## Model of student migration

exm1

Variables and parameters:

- $y_1(t)$: number of students studying option 1
- $y_2(t)$: number of students studying option 2
- $\lambda$: input of fresh students per time interval $\Delta t$
- $a$: effectiveness of communication between the students $y_1$ and $y_2$
- $\beta$: backflow from $y_2$ to $y_1$ ($\beta = 0.1$)
- $d_1$: rate of successful final examination of $y_1$ ($d_1 = 0.1$)
- $d_2$: rate of successful final examination of $y_2$ ($d_2 = 0.3$)
- $\gamma$: portion of graduate students ($\gamma = 0.4$)

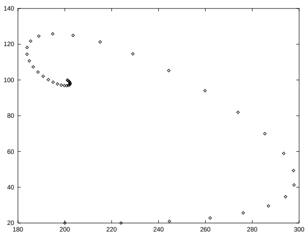
mapping:

$$y_1(t_{j+1}) = y_1(t_j) + \lambda - a(t_j)y_1(t_j)y_2(t_j) + \beta y_2(t_j) - d_1 y_1(t_j)$$
$$y_2(t_{j+1}) = y_2(t_j) + a(t_j)y_1(t_j)y_2(t_j) - \beta y_2(t_j) - d_2 y_2(t_j)$$

bounds:

$$y_1 \geq 0, \quad y_2 \geq 0, \quad a(t)y_1(t)y_2(t) \leq \gamma y_1(t), \quad a(t) \leq a_{\max}$$

Figures:

1 


Phase-diagram of the $(y_1^j, y_2^j)$-values for $j = 0, 1, ..., 40$, $\lambda = 50$, $a = 0.002$.

Comments and explanations:

Original source: J. Scheurle, R. Seydel: A Model of Student Migration. to appear in Int. J. Bifurcation and Chaos

Two fixed points are

$$y_1 = \frac{\lambda}{d_1}, \quad y_2 = 0$$

and

$$y_1 = \frac{\beta + d_2}{a}, \quad y_2 = \frac{1}{d_2}\left(\lambda - \frac{d_1}{a}(\beta + d_2)\right)$$

They are connected by a bifurcation with

$$\lambda a = d_1(\beta + d_2).$$

It is easy to show that the fixed point with $y_2 = 0$ is stable for

$$a\lambda < d_1(\beta + d_2).$$

This defines a threshold value $\lambda_0$ of the number $\lambda$ of entering freshmen, such that for $\lambda < \lambda_0$ the option 2 has no students. The bound is not touched for

$$d_1(\beta + d_2) < a\lambda < d_1(\beta + d_2) + \gamma d_2.$$