Electric Power System (Model of Dobson and Chiang)

exd15

Variables and parameters:

V: magnitude of the load voltage

δ: phase angle of V

δ_m: generator voltage phase angle

ω: rotor speed

Q_1: reactive power demand (bifurcation parameter: $\lambda := Q_1$)

Model equations:

$$\begin{aligned}
\dot{\delta}_m &= \omega \\
M\dot{\omega} &= -\, d_m\omega + P_m - E_m V Y_m \sin(\delta_m - \delta) \\
K_{q\omega}\dot{\delta} &= -\, K_{qv2}V^2 - K_{qv}V + Q(\delta_m, \delta, V) - Q_0 - Q_1 \\
T K_{q\omega} K_{pv} \dot{V} &= K_{p\omega}K_{qv2}V^2 + (K_{p\omega}K_{qv} - K_{q\omega}K_{pv})V \\
&\quad + K_{q\omega}(P(\delta_m, \delta, V) - P_0 - P_1) - K_{p\omega}(Q(\delta_m, \delta, V) - Q_0 - Q_1)
\end{aligned}$$

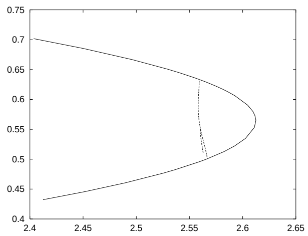
with variables and constants

$$\begin{aligned}
P(\delta_m, \delta, V) &= -\, E_0 V Y_0 \sin(\delta) + E_m V Y_m \sin(\delta_m - \delta) \\
Q(\delta_m, \delta, V) &= E_0 V Y_0 \cos(\delta) + E_m V Y_m \cos(\delta_m - \delta) - (Y_0 + Y_m)V^2
\end{aligned}$$

$M = 0.01464,\ Q_0 = 0.3,\ E_0 = 1.0,\ E_m = 1.05,\ Y_0 = 3.33,\ Y_m = 5.0,$
$K_{p\omega} = 0.4,\ K_{pv} = 0.3,\ K_{q\omega} = -0.03,\ K_{qv} = -2.8,\ K_{qv2} = 2.1,$
$T = 8.5,\ P_0 = 0.6,\ P_1 = 0.0,\ C = 12.0,\ P_m = 1.0,\ d_m = 0.05.$

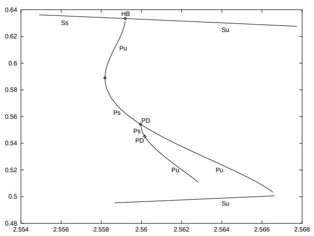
Figures:

1



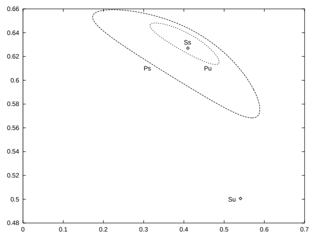
Bifurcation diagram V versus load λ (detail: $\lambda \geq 2.41$, all stationary states and some periodic states; for the latter ones $V_{\min}$ is depicted).

2



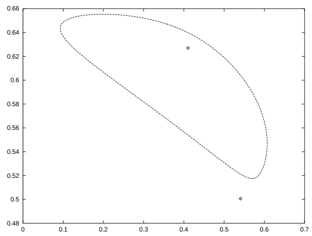
Bifurcation diagram V and $V_{\min}$ versus load λ. detail: $2.555 \leq \lambda \leq 2.568$, all stationary states and some periodic states (S: stationary, P: periodic, s: stable, u: unstable, HB: Hopf bifurcation, PD: periodic doubling)

3



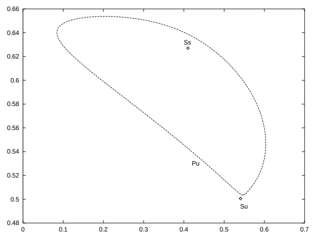
Phase diagram (projection). Horizontal axis: δ_m; vertical axis: V. ($\lambda = 2.55878$)

4



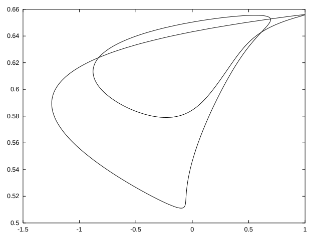
Phase diagram (projection). Horizontal axis: δ_m; vertical axis: V. ($\lambda = 2.5649$)

5



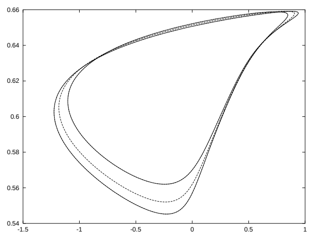
Phase diagram (projection). Horizontal axis: δ_m; vertical axis: V. ($\lambda = 2.566525$)

6



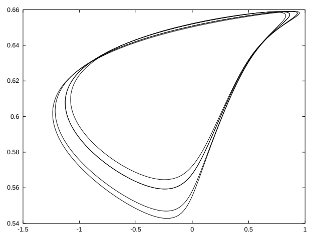
Phase diagram, periodic orbit, unstable $\lambda = 2.5628$, $T = 4.6376$, projection: horizontal axis ω, vertical axis V.

7

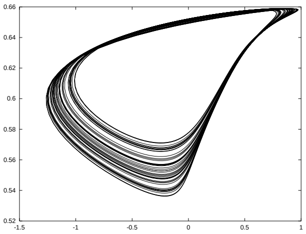


Phase diagram, two periodic orbits for $\lambda = 2.56016$, one stable ($T = 3.802$) and one unstable ($T = 1.913$) projection: horizontal axis ω, vertical axis V.

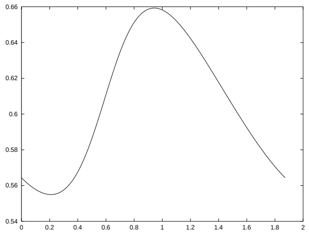
8



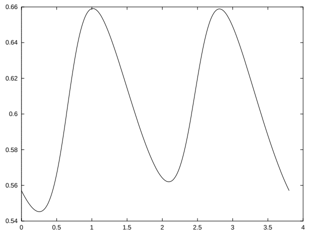
Phase diagram, unstable periodic orbit, $\lambda = 2.5602$, $T = 7.62$, projection: horizontal axis ω, vertical axis V.

9 

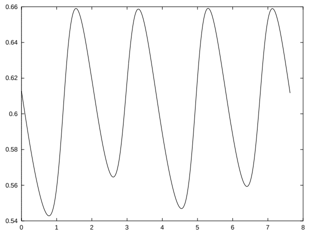
Phase diagram, $\lambda = 2.5603$, $0 \le t \le 100$. projection: horizontal axis ω, vertical axis V.

10 

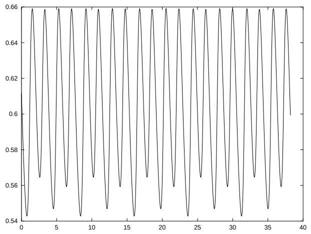
$V(t)$, $\lambda = 2.55988$, $0 \le t \le 1.873$. Periodic orbit: "simple" period.

11 

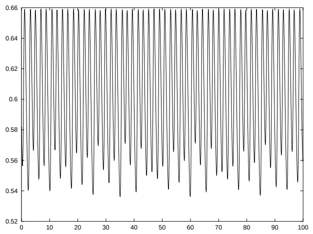
$V(t)$, $\lambda = 2.56016$, $0 \le t \le 3.802$. Periodic orbit: "double" period.

12 

$V(t)$, $\lambda = 2.5602$, $0 \le t \le 7.62$. Periodic orbit: "four-fold" period.

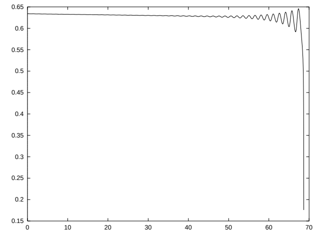
13 

$V(t)$, $\lambda = 2.5602$

14 

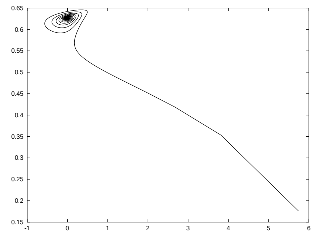
$V(t)$, $\lambda = 2.5603$

15



$V(t)$, with λ varying linearly from $\lambda = 2.5583$ to $\lambda = 2.572$. Collapse of voltage.

16



Phase diagram corresponding to previous figure. Projection to (ω, V)-plane, $0 \leq t \leq 68.67$, with λ varying linearly from $\lambda = 2.5583$ to $\lambda = 2.572$.

Comments and explanations:

Original source of the model: I. Dobson, H.-D. Chiang: Towards a Theory of Voltage Collapse in Electric Power Systems. Systems and Control Letters **13** (1989) 253–262.

The example exhibits a turning point, a Hopf bifurcation, and a period doubling sequence. All of the bifurcations are essential ingredients of a risk analysis for electric power systems. Figures 15 and 16 show a simulation for varying λ (the reactive power demand). The simulation illustrates a collapse of the voltage V of a generator.

For the other figures, the parameter is kept (quasi-)stationary. Figure 1 and 2 show the bifurcation scenario, the other figures show phase portraits and time history for a selection of parameter values. The development of a blue-sky catastrophe is illustrated in Figures 3–5. Figures 10–14 report on the growing complexity of the orbits when λ is passing through the range of period doubling. This leads to chaos, which is illustrated in Figure 9.

For further reference on the risk aspect, see R. Seydel: Assessing Voltage Collapse. Proceedings of a Conference on Risk Analysis. Paris 1998.