Steady-State Brusselator with Diffusion

exb4

Variables and parameters:

x: spatial coordinate, $0 \leq x \leq L$

L : reactor length, $\lambda := L^2$ bifurcation parameter

X, Y: chemicals

$D_1 = 0.0016, \quad D_2 = 0.008$, diffusion constants

$A = 2, \ B = 4.6$

Differential equations:

$$0 = A + X^2Y - BX - X + D_1 \frac{\partial^2 X}{\partial x^2}$$

$$0 = BX - X^2Y + D_2 \frac{\partial^2 Y}{\partial x^2}$$

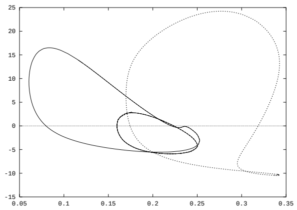
Boundary conditions

$$X = A \qquad \text{for } x = 0, \ x = L$$

$$Y = B/A \qquad \text{for } x = 0, \ x = L$$

Figures:

1



Bifurcation diagram $X'(0)$ versus λ

Comments and explanations:

The system is the same as the one in WOBexa1. Scaling the independent variable according to $t := x/L$ leads to the system of four ODEs of the first order

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= -\lambda[A + y_1^2 y_3 - (B+1)y_1]/D_1 \\ y_3' &= y_4 \\ y_4' &= -\lambda[By_1 - y_1^2 y_3]/D_2 \end{aligned}$$

with boundary conditions

$$y_1(0) = y_1(1) = A, \quad y_3(0) = y_3(1) = B/A.$$

The ordinary differential equations describe the spatial dependence of the two chemicals X and Y along a reactor with length L, $0 \leq x \leq L$. This Brusselator model equation has a great number of solutions ; some are represented in the branching diagram of Figure 1, which depicts $y_2(0) = X'(0)$ versus λ. One nontrivial bifurcation point is found for $\lambda = 0.1698$, $y_2(0) = 6.275$.